# Classical Cryptography

## Polyalphabetic cryptanalysis

Karst Koymans

Informatics Institute

University of Amsterdam

(version 22.3, 2023/02/02 11:37:56 UTC)

Friday, February 17, 2023

# Outline

# The IoC of a polyalphabetic cipher (1)

- We assume that we work with a repeating-key cipher
  - Assume no letters repeat in the key itself
  - Assume the text length is $n$ and the period is $p$
  - For simplicity suppose $p$ divides $n$, so $n = p \cdot q$ and $q = \frac{n}{p}$
- Let $\kappa_r$ be the IoC of random text ($\approx$0.038)
- Let $\kappa_e$ be the IoC of English plaintext ($\approx$0.066)
- If we split up the cryptogram in p columns
  - then each column of size $q$ is monoalphabetic in itself
  - and letters in different columns seem unrelated

# The IoC of a polyalphabetic cipher (2)

So if we pick two different letters from the cryptogram

we expect an index of coincidence of (approximately)

$$\text{IoC} \approx \frac{n(n-q)\kappa_r + n(q-1)\kappa_e}{n(n-1)}$$

or

$$\text{IoC} \approx \frac{n-q}{n-1}\kappa_r + \frac{q-1}{n-1}\kappa_e$$

- For $p = n, q = 1$ this reduces to $\kappa_r$ (random)
- For $p = 1, q = n$ this reduces to $\kappa_e$ (monoalphabetic)

# Outline

# Determination of an unknown period (1)

Solving for p and writing $\kappa_i$ for the IoC we get from the previous estimation

$$p \approx \frac{\kappa_e - \kappa_r}{\kappa_i - \kappa_r + \frac{\kappa_e - \kappa_i}{n}}$$

So if n is large enough this reduces to

$$p \approx \frac{\kappa_e - \kappa_r}{\kappa_i - \kappa_r} \approx \frac{0.028}{\kappa_i - 0.038}$$

# Determination of an unknown period (2)

- The **Kasiski test**
- Look for repetitions of groups of letters in the cryptogram
- See how far they are apart and collect these distances
- Probably the repetitions come from a repetition in the plaintext
- In that case the distance d is a multiple of the period p
- A probable p follows from the consideration of all those d's
- **Charles Babbage** (1791 – 1871) probably invented this method years before **Friedrich Kasiski** (1805–1881) did

# Babbage

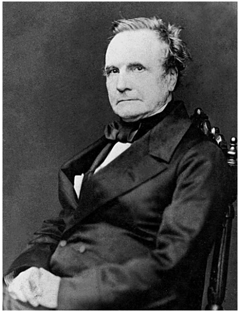


Figure 1: Charles Babbage (1791 – 1871)

Source: https://en.wikipedia.org/wiki/Charles_Babbage

# The Kasiski method

Adapted from slides by Hans van der Meer

# Kasiski method

Until 1863 Vigenère is “le chiffre indéchiffrable”

Then major Friedrich Kasiski publishes
*“Die Geheimschriften und die Dechiffrier-kunst”*
a method to determine the period

uses repetitions in phase with this period

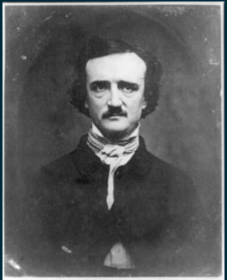
William F. Friedman, Riverbank Publication nr 22, 1920
*The Index of Coincidence and its Application in Cryptography*

# Repetitions

```
pt: EENCURSUSVANHETMATHEMATISCHCENTRUM
 k: STOEIPOESSTOEIPOESSTOEIPOESSTOEIPO   real
ct: WXBGCGGYKNTBLMIAELZXAEBXGGZUXBXZJA

pt: EENCURSUSVANHETMATHEMATISCHCENTRUM
 k: STOEIPOESSTOEIPOESSTOEIPOESSTOEIPO   fake
ct: WXBGCGGYKNTBLMIAELZXAEBXGGZUXBXZJA

pt: EENCURSUSVANHETMATHEMATISCHCENTRUM
 k: STOEIPOESSTOEIPOESSTOEIPOESSTOEIPO   coinci
ct: WXBGCGGYKNTBLMIAELZXAEBXGGZUXBXZJA   dental
```

# Kulp message

```
Ge Jeasgdxv,

Zij gl mw, laam, xzy zmlwhfzek
ejlvdxw kwke tx lbr atgh lbmx
aanu bai Vsmukkss pwn vlwk agh
gnumk wdlnzweg jnbxvv oaeg enwb
zwmgy mo mlw wnbx mw al pnfdcfpkh
wzkex hssf xkiyahul. Mk num yexdm
wbxy sbc hv wyx Phwkgnamcuk?
```

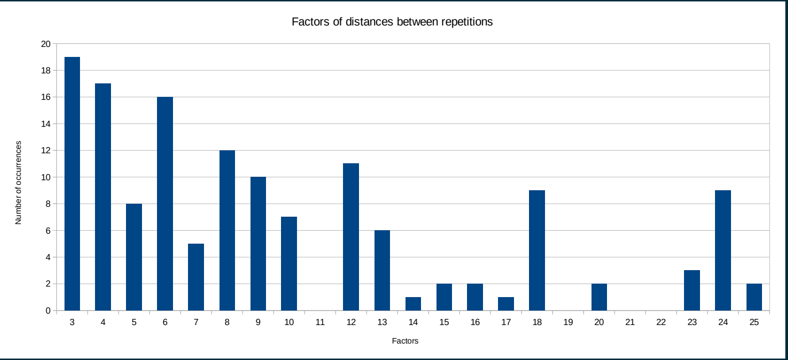


1839 from Kulp, Lewiston, Pennsylvania, USA
to Edgar Allen Poe, ed. Alexander’s Weekly Messenger

Note: jeasgdxv should be ieiasgdxv

# Kasiski analysis

zij gl mw, laam, xzy zmlwhfzek ejlvdxw kwke tx lbr atgh lbmx aanu bai vsmukkss pwn vlwk agh gnumk wdlnzweg jnbxvv oaeg enwb zwmgy mo mlw wnbx mw al pnfdcfpkh wzkex hssf xkiyahul mk num yexdm wbxy sbc hv wyx phwkgnamcuk



# 3 letters = THE ?

zij gl mw, laam, xzy zmlwhfzek ejlvdxw kwke tx
lbr atgh lbmx aanu bai vsmukkss pwn vlwk agh
gnumk wdlnzweg jnbxvv oaeg enwb zwmgy mo mlw
wnbx mw al pnfdcfpkh wzkex hssf xkiyahul mk
num yexdm wbxy sbc hv wyx phwkgnamcuk

XYZ = the → key letters

# Position on period 12

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **I** | **J** | | | | | | | | | | **Z** | ZIJ |
| **Y** | | | | | | | | | | **X** | **Z** | XZY |
| **B** | **R** | | | | | | | | | | **L** | LBR |
| | | **B** | **A** | **I** | | | | | | | | BAI |
| | **P** | **W** | **N** | | | | | | | | | PWN |
| | | | | | | | | **A** | **G** | **H** | | AGH |
| | | | | | | | | | **M** | **L** | **W** | MLW |
| **N** | **U** | **M** | | | | | | | | | | NUM |
| **S** | **B** | **C** | | | | | | | | | | SBC |
| | | | | | **W** | **Y** | **X** | | | | | WYX |

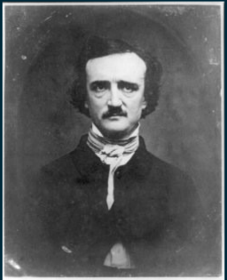
# Key letters

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B | F | | | | | | | | | | G | ZIJ |
| U | | | | | | | | | | E | S | XZY |
| U | N | | | | | | | | | | S | LBR |
| | | I | T | E | | | | | | | | BAI |
| | W | P | J | | | | | | | | | PWN |
| | | | | | | | | H | Z | D | | AGH |
| | | | | | | | | | T | E | S | MLW |
| U | N | I | | | | | | | | | | NUM |
| Z | U | Y | | | | | | | | | | SBC |
| | | | | | D | R | T | | | | | WYX |
| U | N | I | T | E | D | R | T | A | T | E | S | |

Note: The R in DRT should be DST and is one of the many mistakes in the cryptogram

# Kulp message decoded

Mr Alexander,
how ys it, that, the messenger
arrives here at the sace time
with the Saturgay cou rier and
other satuzdao paters when
avco rdidg to the cate it is
publishrd three days previous. Is
the fault witg you or tge
Possmastyrs?



Note the many mistakes (introduced by the editor?)

# Determination of an unknown period (3)

- The $\kappa$ **test**
- Friedman's original application of the theory of coincidence
- This time we look at **two** texts
  - that we compare **character by character**
- We expect coincidences $\kappa_r$ and $\kappa_e$ for respectively two random and two English texts
- The trick is to compare some cryptogram with a **displaced** (shifted, slid) copy of **itself**
- If the displacement is a multiple of the period coincidences rise

# Superimposition

- Knowing the period we can **superimpose** (Dutch: “in diepte leggen”) the cryptogram
- Each column is monoalphabetic
- This makes cryptanalysis easy if the cipher is based

  for instance on a Vigenère with plain alphabet
- Each monoalphabet is then additive and we need only

  one letter for each column to determine it
- Simple letter frequency counts usually suffice

# Outline

# Repeating-key framework for compositions

- Repeating-key polyalphabetic ciphers
- Each monoalphabetic cipher is either
  - Additive
    - So this is a standard Vigenère
  - Affine
    - The first cipher alphabet is mixed up by a decimation

# Keywords of the same length

- Composition gives a similar cipher
- The combined keyword length stays the same
  - Composition of additives stays additive
    - The keyword is the addition of keywords
    - Which makes it somewhat harder-to-guess
  - Composition of affines stays affine
    - The keyword is a linear combination of keywords
    - Also the decimation changes
    - Can you find out the exact formulas?

# Keywords of different lengths

- Let the length of the keywords K and L be $a$ and $b$ respectively
- Let lcm($a$,$b$) be the least common multiple of $a$ and $b$
- Let $a' = \text{lcm}(a, b)/b$ and $b' = \text{lcm}(a, b)/a$
- Reduce this situation to keywords of the same length
  - Consider keywords KK...K ($b'$ times) and LL...L ($a'$ times)
  - This results in two keywords of equal length lcm($a$,$b$)