Classical Cryptography

Polyalphabetic substitution

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(version 22.5, 2023/02/14 13:15:31 UTC)

Tuesday, February 14, 2023

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- Use a changing cipher alphabet (often for each plaintext letter)
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Alberti

Figure 1: Leon Battista Alberti (1404 – 1472)

Source: https://en.wikipedia.org/wiki/Leon_Battista_Alberti

Leon Battista Alberti (1404 – 1472)

- *De Cifris* (On Ciphers)
- Cipher disk
- Regularly change the cipher alphabet
- Communicate a change in ciphertext
- The outer ring is used for plaintext
- The inner ring is used for ciphertext

Trithemius

Figure 2: Johannes Trithemius (1462 – 1516)

Source: https://en.wikipedia.org/wiki/Johannes_Trithemius

Johannes Trithemius (1462 – 1516)

- Tabula recta
  - “proper table”
  - square table
  - letter square
  - tableau
- Progressive system
  - The cipher alphabet changes each letter by taking the next line in the tabula recta

Tabula recta

```
      A B C D E F G H I K L M N O P Q R S T U X Y Z W
    +================================================
 0  | A B C D E F G H I K L M N O P Q R S T U X Y Z W
 1  | B C D E F G H I K L M N O P Q R S T U X Y Z W A
 2  | C D E F G H I K L M N O P Q R S T U X Y Z W A B
 3  | D E F G H I K L M N O P Q R S T U X Y Z W A B C
 4  | E F G H I K L M N O P Q R S T U X Y Z W A B C D
 5  | F G H I K L M N O P Q R S T U X Y Z W A B C D E
 6  | G H I K L M N O P Q R S T U X Y Z W A B C D E F
 7  | H I K L M N O P Q R S T U X Y Z W A B C D E F G
 8  | I K L M N O P Q R S T U X Y Z W A B C D E F G H
 9  | K L M N O P Q R S T U X Y Z W A B C D E F G H I
10  | L M N O P Q R S T U X Y Z W A B C D E F G H I K
11  | M N O P Q R S T U X Y Z W A B C D E F G H I K L
12  | N O P Q R S T U X Y Z W A B C D E F G H I K L M
13  | O P Q R S T U X Y Z W A B C D E F G H I K L M N
14  | P Q R S T U X Y Z W A B C D E F G H I K L M N O
15  | Q R S T U X Y Z W A B C D E F G H I K L M N O P
16  | R S T U X Y Z W A B C D E F G H I K L M N O P Q
17  | S T U X Y Z W A B C D E F G H I K L M N O P Q R
18  | T U X Y Z W A B C D E F G H I K L M N O P Q R S
19  | U X Y Z W A B C D E F G H I K L M N O P Q R S T
20  | X Y Z W A B C D E F G H I K L M N O P Q R S T U
21  | Y Z W A B C D E F G H I K L M N O P Q R S T U X
22  | Z W A B C D E F G H I K L M N O P Q R S T U X Y
23  | W A B C D E F G H I K L M N O P Q R S T U X Y Z
```

Figure 3: Original tabula recta (no J, V; W at end)

```
          A B C D E F G H I J K L M N O P Q R S T U V W X Y Z --> plaintext alphabet
        +=====================================================
     0  | A B C D E F G H I J K L M N O P Q R S T U V W X Y Z \
     1  | B C D E F G H I J K L M N O P Q R S T U V W X Y Z A  |
     2  | C D E F G H I J K L M N O P Q R S T U V W X Y Z A B  |
     3  | D E F G H I J K L M N O P Q R S T U V W X Y Z A B C  |
     4  | E F G H I J K L M N O P Q R S T U V W X Y Z A B C D  |
     5  | F G H I J K L M N O P Q R S T U V W X Y Z A B C D E  |
     6  | G H I J K L M N O P Q R S T U V W X Y Z A B C D E F  |
 p   7  | H I J K L M N O P Q R S T U V W X Y Z A B C D E F G  |
 r   8  | I J K L M N O P Q R S T U V W X Y Z A B C D E F G H  |
 o   9  | J K L M N O P Q R S T U V W X Y Z A B C D E F G H I  |
 g  10  | K L M N O P Q R S T U V W X Y Z A B C D E F G H I J  |
 r  11  | L M N O P Q R S T U V W X Y Z A B C D E F G H I J K  |
 e  12  | M N O P Q R S T U V W X Y Z A B C D E F G H I J K L    \ ciphertext
 s  13  | N O P Q R S T U V W X Y Z A B C D E F G H I J K L M    / alphabets
 s  14  | O P Q R S T U V W X Y Z A B C D E F G H I J K L M N  |
 i  15  | P Q R S T U V W X Y Z A B C D E F G H I J K L M N O  |
 o  16  | Q R S T U V W X Y Z A B C D E F G H I J K L M N O P  |
 n  17  | R S T U V W X Y Z A B C D E F G H I J K L M N O P Q  |
 s  18  | S T U V W X Y Z A B C D E F G H I J K L M N O P Q R  |
    19  | T U V W X Y Z A B C D E F G H I J K L M N O P Q R S  |
    20  | U V W X Y Z A B C D E F G H I J K L M N O P Q R S T  |
    21  | V W X Y Z A B C D E F G H I J K L M N O P Q R S T U  |
    22  | W X Y Z A B C D E F G H I J K L M N O P Q R S T U V  |
    23  | X Y Z A B C D E F G H I J K L M N O P Q R S T U V W  |
    24  | Y Z A B C D E F G H I J K L M N O P Q R S T U V W X  |
    25  | Z A B C D E F G H I J K L M N O P Q R S T U V W X Y /
```

Figure 4: Modern (progressive) tabula recta

Periodic progressive systems

- Normal progression $0, 1, 2, \ldots$ is very regular
  - Its period is 26
- To make things less predictable you can vary the progression
  - A step pattern like $1, 3, 2$ generates the irregular progression $0, 1, 4, 6, 7, 10, 12, \ldots$
  - The progression index (PGI) is $1 + 3 + 2 = 6$ and the progression length (L) is 3
  - Now the period turns out to be $\frac{\text{lcm}(\text{PGI}, 26)}{\text{PGI}} \cdot \text{L} = \frac{\text{lcm}(6, 26)}{6} \cdot 3 = 39$
  - The general formula for an alphabet of size N is $\frac{\text{lcm}(\text{PGI}, \text{N})}{\text{PGI}} \cdot \text{L} = \frac{\text{N}}{\text{gcd}(\text{PGI}, \text{N})} \cdot \text{L}$

Kryha encryption device

- Mechanical device making irregular steps when pushing a lever
  - With 17-steps pattern $7, 6, 7, 5, 6, 7, 6, 8, 6, 10, 5, 6, 5, 7, 6, 5, 9$
  - The period is an impressive $17 \cdot 26 = 442$

Kryha cryptanalysis

- Cryptanalysis by William Friedman and his team
  - William Friedman, Solomon Kullback, Frank Rowlett and Abraham Sinkov
- The challenge given was a 1135 letter cryptogram
- The challenge was broken – without computers – in a mere 2 hours and 41 minutes

Giovan Battista Bellaso (1505 – ca 1575)

- “Forgotten by history”
- Introduced the keyed polyalphabet
  - Repeating-key cipher
  - Later named after Blaise de **Vigenère**
- Used reciprocal alphabets
  - Makes encryption and decryption identical operations
  - Later named after Francis **Beaufort**

Vigenère

Figure 5: Blaise de Vigenère (1523 – 1596)

Source: https://en.wikipedia.org/wiki/Blaise_de_Vigenère

Blaise de Vigenère (1523 – 1596)

- Used Bellaso’s ideas
- Combined the following ideas
  - Tabula recta (now called Vigenère square)
  - Repeating-key cipher
- Plaintext letters are along the top of the diagram
- Ciphertext letters inside the table
- Key letters are along the left side of the diagram
  - A key letter equals the first letter of the cipher alphabet

```
                             plaintext

       A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
     +====================================================
   A | A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
   B | B C D E F G H I J K L M N O P Q R S T U V W X Y Z A
   C | C D E F G H I J K L M N O P Q R S T U V W X Y Z A B
   D | D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
   E | E F G H I J K L M N O P Q R S T U V W X Y Z A B C D
   F | F G H I J K L M N O P Q R S T U V W X Y Z A B C D E
   G | G H I J K L M N O P Q R S T U V W X Y Z A B C D E F
   H | H I J K L M N O P Q R S T U V W X Y Z A B C D E F G
 k I | I J K L M N O P Q R S T U V W X Y Z A B C D E F G H
 e J | J K L M N O P Q R S T U V W X Y Z A B C D E F G H I
 y K | K L M N O P Q R S T U V W X Y Z A B C D E F G H I J
   L | L M N O P Q R S T U V W X Y Z A B C D E F G H I J K
 l M | M N O P Q R S T U V W X Y Z A B C D E F G H I J K L
 e N | N O P Q R S T U V W X Y Z A B C D E F G H I J K L M
 t O | O P Q R S T U V W X Y Z A B C D E F G H I J K L M N
 t P | P Q R S T U V W X Y Z A B C D E F G H I J K L M N O
 e Q | Q R S T U V W X Y Z A B C D E F G H I J K L M N O P
 r R | R S T U V W X Y Z A B C D E F G H I J K L M N O P Q
   S | S T U V W X Y Z A B C D E F G H I J K L M N O P Q R
   T | T U V W X Y Z A B C D E F G H I J K L M N O P Q R S
   U | U V W X Y Z A B C D E F G H I J K L M N O P Q R S T
   V | V W X Y Z A B C D E F G H I J K L M N O P Q R S T U
   W | W X Y Z A B C D E F G H I J K L M N O P Q R S T U V
   X | X Y Z A B C D E F G H I J K L M N O P Q R S T U V W
   Y | Y Z A B C D E F G H I J K L M N O P Q R S T U V W X
   Z | Z A B C D E F G H I J K L M N O P Q R S T U V W X Y
```

Figure 6: Vigenère table (modern encoding)

Mathematical formulation of Vigenère's encryption

- Let $P = P_0 P_1 \dots P_{n-1}$ be the plaintext
- Let $K = K_0 K_1 \dots K_{p-1}$ be the key with **period p**
- Then the cryptogram $C = C_0 C_1 \dots C_{n-1}$ is given by
  - $C_i = \mathcal{E}_i(P_i) = P_i + K_{i \pmod p} \pmod{26}$
- For decryption we conclude
  - $P_i = \mathcal{D}_i(C_i) = C_i - K_{i \pmod p} \pmod{26}$
- Exchanging encryption and decryption is called "Variant Vigenère"

More room for confusion

- Assume we want to keep the simple mathematical relationship between plaintext letter and cryptogram letter: $C = P + K$
- Also assume we want to use legacy encoding
- The only way this works is by using an **alternative** Vigenère
- This non-standard table is what is used in "The Mathematics of Secrets"
- In this case the key letters are not the first elements of the cipher alphabet

```
                     plaintext

       A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
     +====================================================
   A | B C D E F G H I J K L M N O P Q R S T U V W X Y Z A
   B | C D E F G H I J K L M N O P Q R S T U V W X Y Z A B
   C | D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
   D | E F G H I J K L M N O P Q R S T U V W X Y Z A B C D
   E | F G H I J K L M N O P Q R S T U V W X Y Z A B C D E
   F | G H I J K L M N O P Q R S T U V W X Y Z A B C D E F
   G | H I J K L M N O P Q R S T U V W X Y Z A B C D E F G
   H | I J K L M N O P Q R S T U V W X Y Z A B C D E F G H
k  I | J K L M N O P Q R S T U V W X Y Z A B C D E F G H I
e  J | K L M N O P Q R S T U V W X Y Z A B C D E F G H I J
y  K | L M N O P Q R S T U V W X Y Z A B C D E F G H I J K
   L | M N O P Q R S T U V W X Y Z A B C D E F G H I J K L
l  M | N O P Q R S T U V W X Y Z A B C D E F G H I J K L M
e  N | O P Q R S T U V W X Y Z A B C D E F G H I J K L M N
t  O | P Q R S T U V W X Y Z A B C D E F G H I J K L M N O
t  P | Q R S T U V W X Y Z A B C D E F G H I J K L M N O P
e  Q | R S T U V W X Y Z A B C D E F G H I J K L M N O P Q
r  R | S T U V W X Y Z A B C D E F G H I J K L M N O P Q R
   S | T U V W X Y Z A B C D E F G H I J K L M N O P Q R S
   T | U V W X Y Z A B C D E F G H I J K L M N O P Q R S T
   U | V W X Y Z A B C D E F G H I J K L M N O P Q R S T U
   V | W X Y Z A B C D E F G H I J K L M N O P Q R S T U V
   W | X Y Z A B C D E F G H I J K L M N O P Q R S T U V W
   X | Y Z A B C D E F G H I J K L M N O P Q R S T U V W X
   Y | Z A B C D E F G H I J K L M N O P Q R S T U V W X Y
   Z | A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
```

Figure 7: Alternative Vigenère table (legacy encoding; used in book)

Beaufort

Figure 8: Francis Beaufort (1774 – 1857)

Source: https://en.wikipedia.org/wiki/Francis_Beaufort

Francis Beaufort (1774 – 1857)

- Changes Vigenère square by starting with a mixed cipher alphabet
  - Which is a Caesar (key = 1) shift of the atbash cipher
  - Or if you want the atbash of a Caesar (key = -1) shift
- In modern encoding the Beaufort starting cipher alphabetcan also be described simply as a multiplicative cipher with factor -1
- In legacy encoding the Beaufort starting cipher alphabet
  - must be described by a more complicated affine cipher with
    - factor -1
    - additive 2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B
B	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C
C	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D
D	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E
E	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F
F	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G
G	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H
H	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I
I	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J
J	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K
K	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L
L	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M
M	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O	N
N	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P	O
O	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q	P
P	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R	Q
Q	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S	R
R	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T	S
S	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U	T
T	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V	U
U	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W	V
V	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X	W
W	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y	X
X	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z	Y
Y	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A	Z
Z	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A

Figure 9: Beaufort table

Mathematical formulation of Beaufort's encryption

- Let $P = P_0 P_1 \dots P_{n-1}$ be the plaintext (in modern encoding)
- Let $K = K_0 K_1 \dots K_{p-1}$ be the key with **period p**
- Then the cryptogram $C = C_0 C_1 \dots C_{n-1}$ is given by
  - $C_i = \mathcal{E}_i(P_i) = -P_i + K_{i \pmod p} \pmod{26}$
- For decryption we conclude
  - $P_i = \mathcal{D}_i(C_i) = -C_i + K_{i \pmod p} \pmod{26}$
- Now we clearly see the symmetric role of encryption and decryption
  - $P_i + C_i = C_i + P_i = K_{i \pmod p} \pmod{26}$

della Porta

Figure 10: Giambattista della Porta (ca 1535 – 1615)

Source: https://en.wikipedia.org/wiki/Giambattista_della_Porta

Giambattista della Porta (ca 1535 – 1615)

- Introduced the first digraph substitution
  - *De furtivis Literarum Notis* (1563)
    - His scientific work on cryptography
- Introduced another polyalphabetic cipher based on a reduced size table
  - Porta's reduced table

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
B	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
C	O	P	Q	R	S	T	U	V	W	X	Y	Z	N	M	A	B	C	D	E	F	G	H	I	J	K	L
D	O	P	Q	R	S	T	U	V	W	X	Y	Z	N	M	A	B	C	D	E	F	G	H	I	J	K	L
E	P	Q	R	S	T	U	V	W	X	Y	Z	N	O	L	M	A	B	C	D	E	F	G	H	I	J	K
F	P	Q	R	S	T	U	V	W	X	Y	Z	N	O	L	M	A	B	C	D	E	F	G	H	I	J	K
G	Q	R	S	T	U	V	W	X	Y	Z	N	O	P	K	L	M	A	B	C	D	E	F	G	H	I	J
H	Q	R	S	T	U	V	W	X	Y	Z	N	O	P	K	L	M	A	B	C	D	E	F	G	H	I	J
I	R	S	T	U	V	W	X	Y	Z	N	O	P	Q	J	K	L	M	A	B	C	D	E	F	G	H	I
J	R	S	T	U	V	W	X	Y	Z	N	O	P	Q	J	K	L	M	A	B	C	D	E	F	G	H	I
K	S	T	U	V	W	X	Y	Z	N	O	P	Q	R	I	J	K	L	M	A	B	C	D	E	F	G	H
L	S	T	U	V	W	X	Y	Z	N	O	P	Q	R	I	J	K	L	M	A	B	C	D	E	F	G	H
M	T	U	V	W	X	Y	Z	N	O	P	Q	R	S	H	I	J	K	L	M	A	B	C	D	E	F	G
N	T	U	V	W	X	Y	Z	N	O	P	Q	R	S	H	I	J	K	L	M	A	B	C	D	E	F	G
O	U	V	W	X	Y	Z	N	O	P	Q	R	S	T	G	H	I	J	K	L	M	A	B	C	D	E	F
P	U	V	W	X	Y	Z	N	O	P	Q	R	S	T	G	H	I	J	K	L	M	A	B	C	D	E	F
Q	V	W	X	Y	Z	N	O	P	Q	R	S	T	U	F	G	H	I	J	K	L	M	A	B	C	D	E
R	V	W	X	Y	Z	N	O	P	Q	R	S	T	U	F	G	H	I	J	K	L	M	A	B	C	D	E
S	W	X	Y	Z	N	O	P	Q	R	S	T	U	V	E	F	G	H	I	J	K	L	M	A	B	C	D
T	W	X	Y	Z	N	O	P	Q	R	S	T	U	V	E	F	G	H	I	J	K	L	M	A	B	C	D
U	X	Y	Z	N	O	P	Q	R	S	T	U	V	W	D	E	F	G	H	I	J	K	L	M	A	B	C
V	X	Y	Z	N	O	P	Q	R	S	T	U	V	W	D	E	F	G	H	I	J	K	L	M	A	B	C
W	Y	Z	N	O	P	Q	R	S	T	U	V	W	X	C	D	E	F	G	H	I	J	K	L	M	A	B
X	Y	Z	N	O	P	Q	R	S	T	U	V	W	X	C	D	E	F	G	H	I	J	K	L	M	A	B
Y	Z	N	O	P	Q	R	S	T	U	V	W	X	Y	B	C	D	E	F	G	H	I	J	K	L	M	A
Z	Z	N	O	P	Q	R	S	T	U	V	W	X	Y	B	C	D	E	F	G	H	I	J	K	L	M	A

Figure 11: Full Porta table

Reduced Porta table

	A	B	C	D	E	F	G	H	I	J	K	L	M
AB	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
CD	O	P	Q	R	S	T	U	V	W	X	Y	Z	N
EF	P	Q	R	S	T	U	V	W	X	Y	Z	N	O
GH	Q	R	S	T	U	V	W	X	Y	Z	N	O	P
IJ	R	S	T	U	V	W	X	Y	Z	N	O	P	Q
KL	S	T	U	V	W	X	Y	Z	N	O	P	Q	R
MN	T	U	V	W	X	Y	Z	N	O	P	Q	R	S
OP	U	V	W	X	Y	Z	N	O	P	Q	R	S	T
QR	V	W	X	Y	Z	N	O	P	Q	R	S	T	U
ST	W	X	Y	Z	N	O	P	Q	R	S	T	U	V
UV	X	Y	Z	N	O	P	Q	R	S	T	U	V	W
WX	Y	Z	N	O	P	Q	R	S	T	U	V	W	X
YZ	Z	N	O	P	Q	R	S	T	U	V	W	X	Y

Figure 12: Reduced Porta table

	P	L	A	I	N	M	X	E	D	U	B	C	F	G	H	J	K	O	Q	R	S	T	V	W	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Figure 13: “Plain mixed up”-table

```
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
  +====================================================
A | C I P H E R M X D U A B F G J K L N O Q S T V W Y Z
B | I P H E R M X D U A B F G J K L N O Q S T V W Y Z C
C | P H E R M X D U A B F G J K L N O Q S T V W Y Z C I
D | H E R M X D U A B F G J K L N O Q S T V W Y Z C I P
E | E R M X D U A B F G J K L N O Q S T V W Y Z C I P H
F | R M X D U A B F G J K L N O Q S T V W Y Z C I P H E
G | M X D U A B F G J K L N O Q S T V W Y Z C I P H E R
H | X D U A B F G J K L N O Q S T V W Y Z C I P H E R M
I | D U A B F G J K L N O Q S T V W Y Z C I P H E R M X
J | U A B F G J K L N O Q S T V W Y Z C I P H E R M X D
K | A B F G J K L N O Q S T V W Y Z C I P H E R M X D U
L | B F G J K L N O Q S T V W Y Z C I P H E R M X D U A
M | F G J K L N O Q S T V W Y Z C I P H E R M X D U A B
N | G J K L N O Q S T V W Y Z C I P H E R M X D U A B F
O | J K L N O Q S T V W Y Z C I P H E R M X D U A B F G
P | K L N O Q S T V W Y Z C I P H E R M X D U A B F G J
Q | L N O Q S T V W Y Z C I P H E R M X D U A B F G J K
R | N O Q S T V W Y Z C I P H E R M X D U A B F G J K L
S | O Q S T V W Y Z C I P H E R M X D U A B F G J K L N
T | Q S T V W Y Z C I P H E R M X D U A B F G J K L N O
U | S T V W Y Z C I P H E R M X D U A B F G J K L N O Q
V | T V W Y Z C I P H E R M X D U A B F G J K L N O Q S
W | V W Y Z C I P H E R M X D U A B F G J K L N O Q S T
X | W Y Z C I P H E R M X D U A B F G J K L N O Q S T V
Y | Y Z C I P H E R M X D U A B F G J K L N O Q S T V W
Z | Z C I P H E R M X D U A B F G J K L N O Q S T V W Y
```

Figure 14: “Cipher mixed up”-table

```
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
  +====================================================
C | C I P H E R M X D U A B F G J K L N O Q S T V W Y Z
I | I P H E R M X D U A B F G J K L N O Q S T V W Y Z C
P | P H E R M X D U A B F G J K L N O Q S T V W Y Z C I
H | H E R M X D U A B F G J K L N O Q S T V W Y Z C I P
E | E R M X D U A B F G J K L N O Q S T V W Y Z C I P H
R | R M X D U A B F G J K L N O Q S T V W Y Z C I P H E
M | M X D U A B F G J K L N O Q S T V W Y Z C I P H E R
X | X D U A B F G J K L N O Q S T V W Y Z C I P H E R M
D | D U A B F G J K L N O Q S T V W Y Z C I P H E R M X
U | U A B F G J K L N O Q S T V W Y Z C I P H E R M X D
A | A B F G J K L N O Q S T V W Y Z C I P H E R M X D U
B | B F G J K L N O Q S T V W Y Z C I P H E R M X D U A
F | F G J K L N O Q S T V W Y Z C I P H E R M X D U A B
G | G J K L N O Q S T V W Y Z C I P H E R M X D U A B F
J | J K L N O Q S T V W Y Z C I P H E R M X D U A B F G
K | K L N O Q S T V W Y Z C I P H E R M X D U A B F G J
L | L N O Q S T V W Y Z C I P H E R M X D U A B F G J K
N | N O Q S T V W Y Z C I P H E R M X D U A B F G J K L
O | O Q S T V W Y Z C I P H E R M X D U A B F G J K L N
Q | Q S T V W Y Z C I P H E R M X D U A B F G J K L N O
S | S T V W Y Z C I P H E R M X D U A B F G J K L N O Q
T | T V W Y Z C I P H E R M X D U A B F G J K L N O Q S
V | V W Y Z C I P H E R M X D U A B F G J K L N O Q S T
W | W Y Z C I P H E R M X D U A B F G J K L N O Q S T V
Y | Y Z C I P H E R M X D U A B F G J K L N O Q S T V W
Z | Z C I P H E R M X D U A B F G J K L N O Q S T V W Y
```

Figure 15: “Cipher and key mixed up”-table

```
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
  +====================================================
A | A B F G J K L N O Q S T V W Y Z C I P H E R M X D U
B | B F G J K L N O Q S T V W Y Z C I P H E R M X D U A
C | C I P H E R M X D U A B F G J K L N O Q S T V W Y Z
D | D U A B F G J K L N O Q S T V W Y Z C I P H E R M X
E | E R M X D U A B F G J K L N O Q S T V W Y Z C I P H
F | F G J K L N O Q S T V W Y Z C I P H E R M X D U A B
G | G J K L N O Q S T V W Y Z C I P H E R M X D U A B F
H | H E R M X D U A B F G J K L N O Q S T V W Y Z C I P
I | I P H E R M X D U A B F G J K L N O Q S T V W Y Z C
J | J K L N O Q S T V W Y Z C I P H E R M X D U A B F G
K | K L N O Q S T V W Y Z C I P H E R M X D U A B F G J
L | L N O Q S T V W Y Z C I P H E R M X D U A B F G J K
M | M X D U A B F G J K L N O Q S T V W Y Z C I P H E R
N | N O Q S T V W Y Z C I P H E R M X D U A B F G J K L
O | O Q S T V W Y Z C I P H E R M X D U A B F G J K L N
P | P H E R M X D U A B F G J K L N O Q S T V W Y Z C I
Q | Q S T V W Y Z C I P H E R M X D U A B F G J K L N O
R | R M X D U A B F G J K L N O Q S T V W Y Z C I P H E
S | S T V W Y Z C I P H E R M X D U A B F G J K L N O Q
T | T V W Y Z C I P H E R M X D U A B F G J K L N O Q S
U | U A B F G J K L N O Q S T V W Y Z C I P H E R M X D
V | V W Y Z C I P H E R M X D U A B F G J K L N O Q S T
W | W Y Z C I P H E R M X D U A B F G J K L N O Q S T V
X | X D U A B F G J K L N O Q S T V W Y Z C I P H E R M
Y | Y Z C I P H E R M X D U A B F G J K L N O Q S T V W
Z | Z C I P H E R M X D U A B F G J K L N O Q S T V W Y
```

Figure 16: “Cipher and key mixed up (sorted)”-table

```
    P L A I N M X E D U B C F G H J K O Q R S T V W Y Z
  +====================================================
K | C I P H E R M X D U A B F G J K L N O Q S T V W Y Z
E | I P H E R M X D U A B F G J K L N O Q S T V W Y Z C
Y | P H E R M X D U A B F G J K L N O Q S T V W Y Z C I
M | H E R M X D U A B F G J K L N O Q S T V W Y Z C I P
I | E R M X D U A B F G J K L N O Q S T V W Y Z C I P H
X | R M X D U A B F G J K L N O Q S T V W Y Z C I P H E
D | M X D U A B F G J K L N O Q S T V W Y Z C I P H E R
U | X D U A B F G J K L N O Q S T V W Y Z C I P H E R M
P | D U A B F G J K L N O Q S T V W Y Z C I P H E R M X
A | U A B F G J K L N O Q S T V W Y Z C I P H E R M X D
B | A B F G J K L N O Q S T V W Y Z C I P H E R M X D U
C | B F G J K L N O Q S T V W Y Z C I P H E R M X D U A
F | F G J K L N O Q S T V W Y Z C I P H E R M X D U A B
G | G J K L N O Q S T V W Y Z C I P H E R M X D U A B F
H | J K L N O Q S T V W Y Z C I P H E R M X D U A B F G
J | K L N O Q S T V W Y Z C I P H E R M X D U A B F G J
L | L N O Q S T V W Y Z C I P H E R M X D U A B F G J K
N | N O Q S T V W Y Z C I P H E R M X D U A B F G J K L
O | O Q S T V W Y Z C I P H E R M X D U A B F G J K L N
Q | Q S T V W Y Z C I P H E R M X D U A B F G J K L N O
R | S T V W Y Z C I P H E R M X D U A B F G J K L N O Q
S | T V W Y Z C I P H E R M X D U A B F G J K L N O Q S
T | V W Y Z C I P H E R M X D U A B F G J K L N O Q S T
V | W Y Z C I P H E R M X D U A B F G J K L N O Q S T V
W | Y Z C I P H E R M X D U A B F G J K L N O Q S T V W
Z | Z C I P H E R M X D U A B F G J K L N O Q S T V W Y
```

Figure 17: “Plain, cipher and key mixed up”-table

```
    P L A I N M X E D U B C F G H J K O Q R S T V W Y Z
  +====================================================
A | U A B F G J K L N O Q S T V W Y Z C I P H E R M X D
B | A B F G J K L N O Q S T V W Y Z C I P H E R M X D U
C | B F G J K L N O Q S T V W Y Z C I P H E R M X D U A
D | M X D U A B F G J K L N O Q S T V W Y Z C I P H E R
E | I P H E R M X D U A B F G J K L N O Q S T V W Y Z C
F | F G J K L N O Q S T V W Y Z C I P H E R M X D U A B
G | G J K L N O Q S T V W Y Z C I P H E R M X D U A B F
H | J K L N O Q S T V W Y Z C I P H E R M X D U A B F G
I | E R M X D U A B F G J K L N O Q S T V W Y Z C I P H
J | K L N O Q S T V W Y Z C I P H E R M X D U A B F G J
K | C I P H E R M X D U A B F G J K L N O Q S T V W Y Z
L | L N O Q S T V W Y Z C I P H E R M X D U A B F G J K
M | H E R M X D U A B F G J K L N O Q S T V W Y Z C I P
N | N O Q S T V W Y Z C I P H E R M X D U A B F G J K L
O | O Q S T V W Y Z C I P H E R M X D U A B F G J K L N
P | D U A B F G J K L N O Q S T V W Y Z C I P H E R M X
Q | Q S T V W Y Z C I P H E R M X D U A B F G J K L N O
R | S T V W Y Z C I P H E R M X D U A B F G J K L N O Q
S | T V W Y Z C I P H E R M X D U A B F G J K L N O Q S
T | V W Y Z C I P H E R M X D U A B F G J K L N O Q S T
U | X D U A B F G J K L N O Q S T V W Y Z C I P H E R M
V | W Y Z C I P H E R M X D U A B F G J K L N O Q S T V
W | Y Z C I P H E R M X D U A B F G J K L N O Q S T V W
X | R M X D U A B F G J K L N O Q S T V W Y Z C I P H E
Y | P H E R M X D U A B F G J K L N O Q S T V W Y Z C I
Z | Z C I P H E R M X D U A B F G J K L N O Q S T V W Y
```

Figure 18: “Plain, cipher and key mixed up (sorted)”-table

```
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
  +====================================================
A | B Q S N L T V W F Y Z A J G C U I P H E O R M K X D
B | F S T O N V W Y G Z C B K J I A P H E R Q M X L D U
C | G T V Q O W Y Z J C I F L K P B H E R M S X D N U A
D | D L N J G O Q S U T V X B A W M Y Z C I K P H F E R
E | H B F U D G J K E L N P M R O I Q S T V A W Y X Z C
F | J V W S Q Y Z C K I P G N L H F E R M X T D U O A B
G | K W Y T S Z C I L P H J O N E G R M X D V U A Q B F
H | L Y Z V T C I P N H E K Q O R J M X D U W A B S F G
I | M J K F B L N O X Q S R U D T E V W Y Z G C I A P H
J | N Z C W V I P H O E R L S Q M K X D U A Y B F T G J
K | P A B D X F G J H K L I R E N C O Q S T U V W M Y Z
L | O C I Y W P H E Q R M N T S X L D U A B Z F G V J K
M | R G J B A K L N M O Q E D X S H T V W Y F Z C U I P
N | Q I P Z Y H E R S M X O V T D N U A B F C G J W K L
O | S P H C Z E R M T X D Q W V U O A B F G I J K Y L N
P | A O Q L K S T V B W Y U G F Z D C I P H N E R J M X
Q | T H E I C R M X V D U S Y W A Q B F G J P K L Z N O
R | V E R P I M X D W U A T Z Y B S F G J K H L N C O Q
S | W R M H P X D U Y A B V C Z F T G J K L E N O I Q S
T | Y M X E H D U A Z B F W I C G V J K L N R O Q P S T
U | U N O K J Q S T A V W D F B Y X Z C I P L H E G R M
V | Z X D R E U A B C F G Y P I J W K L N O M Q S H T V
W | C D U M R A B F I G J Z H P K Y L N O Q X S T E V W
X | X K L G F N O Q D S T M A U V R W Y Z C J I P B H E
Y | E F G A U J K L R N O H X M Q P S T V W B Y Z D C I
Z | I U A X M B F G P J K C E H L Z N O Q S D T V R W Y
```

Figure 19: “Plain, cipher and key mixed up (all sorted)”-table

```
    S A M E I X D B C F G H J K L N O P Q R T U V W Y Z
  +====================================================
S | S Z Y W V U T R Q P O N L K J H G F C B D X I E M A
A | A S Z Y W V U T R Q P O N L K J H G F C B D X I E M
M | M A S Z Y W V U T R Q P O N L K J H G F C B D X I E
E | E M A S Z Y W V U T R Q P O N L K J H G F C B D X I
I | I E M A S Z Y W V U T R Q P O N L K J H G F C B D X
X | X I E M A S Z Y W V U T R Q P O N L K J H G F C B D
D | D X I E M A S Z Y W V U T R Q P O N L K J H G F C B
B | B D X I E M A S Z Y W V U T R Q P O N L K J H G F C
C | C B D X I E M A S Z Y W V U T R Q P O N L K J H G F
F | F C B D X I E M A S Z Y W V U T R Q P O N L K J H G
G | G F C B D X I E M A S Z Y W V U T R Q P O N L K J H
H | H G F C B D X I E M A S Z Y W V U T R Q P O N L K J
J | J H G F C B D X I E M A S Z Y W V U T R Q P O N L K
K | K J H G F C B D X I E M A S Z Y W V U T R Q P O N L
L | L K J H G F C B D X I E M A S Z Y W V U T R Q P O N
N | N L K J H G F C B D X I E M A S Z Y W V U T R Q P O
O | O N L K J H G F C B D X I E M A S Z Y W V U T R Q P
P | P O N L K J H G F C B D X I E M A S Z Y W V U T R Q
Q | Q P O N L K J H G F C B D X I E M A S Z Y W V U T R
R | R Q P O N L K J H G F C B D X I E M A S Z Y W V U T
T | T R Q P O N L K J H G F C B D X I E M A S Z Y W V U
U | U T R Q P O N L K J H G F C B D X I E M A S Z Y W V
V | V U T R Q P O N L K J H G F C B D X I E M A S Z Y W
W | W V U T R Q P O N L K J H G F C B D X I E M A S Z Y
Y | Y W V U T R Q P O N L K J H G F C B D X I E M A S Z
Z | Z Y W V U T R Q P O N L K J H G F C B D X I E M A S
```

Figure 20: “Same mixed (Beaufort-style)” table

```
    A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
  +====================================================
A | S T R U Y Q P O W N L K Z J H G F C A B D X I V E M
B | D S Z A I Y W V E U T R X Q P O N L B K J H G M F C
C | B A S M X Z Y W I V U T D R Q P O N C L K J H E G F
D | X Z Y S E W V U M T R Q I P O N L K D J H G F A C B
E | M V U W S T R Q Z P O N A L K J H G E F C B D Y X I
F | C M A E D S Z Y X W V U B T R Q P O F N L K J I H G
G | F E M I B A S Z D Y W V C U T R Q P G O N L K X J H
H | G I E X C M A S B Z Y W F V U T R Q H P O N L D K J
I | E W V Y A U T R S Q P O M N L K J H I G F C B Z D X
J | H X I D F E M A C S Z Y G W V U T R J Q P O N B L K
K | J D X B G I E M F A S Z H Y W V U T K R Q P O C N L
L | K B D C H X I E G M A S J Z Y W V U L T R Q P F O N
M | A U T V Z R Q P Y O N L S K J H G F M C B D X W I E
N | L C B F J D X I H E M A K S Z Y W V N U T R Q G P O
O | N F C G K B D X J I E M L A S Z Y W O V U T R H Q P
P | O G F H L C B D K X I E N M A S Z Y P W V U T J R Q
Q | P H G J N F C B L D X I O E M A S Z Q Y W V U K T R
R | Q J H K O G F C N B D X P I E M A S R Z Y W V L U T
S | Z R Q T W P O N V L K J Y H G F C B S D X I E U M A
T | R K J L P H G F O C B D Q X I E M A T S Z Y W N V U
U | T L K N Q J H G P F C B R D X I E M U A S Z Y O W V
V | U N L O R K J H Q G F C T B D X I E V M A S Z P Y W
W | V O N P T L K J R H G F U C B D X I W E M A S Q Z Y
X | I Y W Z M V U T A R Q P E O N L K J X H G F C S B D
Y | W P O Q U N L K T J H G V F C B D X Y I E M A R S Z
Z | Y Q P R V O N L U K J H W G F C B D Z X I E M T A S
```

Figure 21: “Same mixed (Beaufort style and sorted)”-table

Multiplex systems (1): Alphabet strips

- Choose a set of alphabet strips from a given collection
- Each strip contains (two copies of) a permutation of the alphabet
- Put the plaintext inside one of the columns
- Read off the ciphertext from any other column
  - Each of the other 25 columns is called a **generatrix**

Alphabet strip example

```
               plain           crypto
               v               w
11 A L T M S X V Q P N O H U W D I Z Y C G K R F B E J
 3 C Z I N X F Y Q R T V W L A D K O M J U B G E P H S
23 J C P G B Z A X K W R E V D T U F O Y H M L S I Q N
12 V E W O A M N F L H Q G C U J T B Y P Z K X I S R D
 7 V R O G S Y D U L C F M Q T W A H X J E Z B N I K P
       w       v
11 M S X V Q P N O H U W D I Z Y C G K R F B E J A L T
 3 Q R T V W L A D K O M J U B G E P H S C Z I N X F Y
23 X K W R E V D T U F O Y H M L S I Q N J C P G B Z A
12 X I S R D V E W O A M N F L H Q G C U J T B Y P Z K
 7 B N I K P V R O G S Y D U L C F M Q T W A H X J E Z
               v       w
11 F B E J A L T M S X V Q P N O H U W D I Z Y C G K R
 3 F Y Q R T V W L A D K O M J U B G E P H S C Z I N X
23 J C P G B Z A X K W R E V D T U F O Y H M L S I Q N
12 H Q G C U J T B Y P Z K X I S R D V E W O A M N F L
 7 T W A H X J E Z B N I K P V R O G S Y D U L C F M Q
               v                   w
11 I Z Y C G K R F B E J A L T M S X V Q P N O H U W D
```

Figure 22: Encryption of VYAND NADERT WATER into DDTJW XTWSI VKRZI X
Source: Syllabus Hans van der Meer

Alphabet strips (M-138-A)

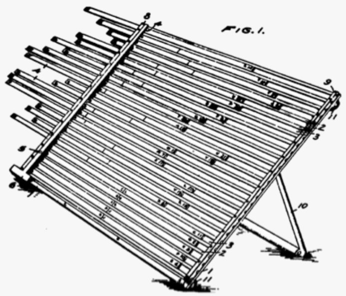
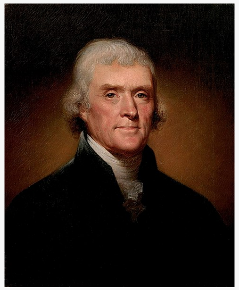


Figure 23: William Friedman's alphabet strips device

Source: Syllabus Hans van der Meer

Jefferson and Bazeries



Thomas Jefferson

Étienne Bazeries

https://en.wikipedia.org/wiki/Thomas_Jefferson

http://www.bibmath.net/crypto/complements/images/bazeries.jpg

Multiplex systems (2): Jefferson disk (M-94)

- This is based on the same idea as the alphabet strips
- The alphabets are circumscribed on wheels mounted on a cylinder
- It is also called a Bazeries cylinder

Source: Syllabus Hans van der Meer

Rotor based systems

- Similar to a progressive system based on a mixed cipher alphabet
- The difference is that it also has a "regressive" component
- In fact the next cipher alphabet is a **conjugation**
  of the current cipher alphabet with a "Caesar 1" cipher
- Let R be the (arbitrary) rotor permutation and C an additive permutation with addition 1
- Then after k rotation steps the permutation is given by

$$R_k = C^{-k} \circ R \circ C^{k}$$