Classical Cryptography

Basics: transpositions

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Table of Contents

Theoretic considerations
- Permutations

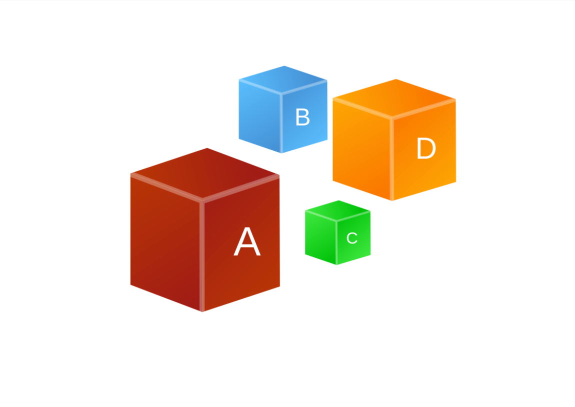
Framework for cryptography

Transpositions
- Geometric/Route ciphers
- Permutation ciphers
- Historic examples

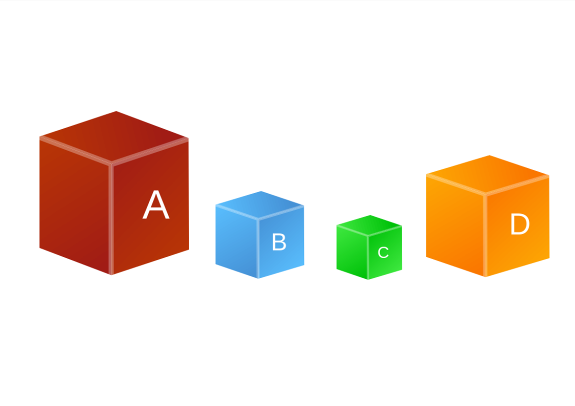
Substitutions versus Transpositions

- Substitutions
  - transform objects into or replace objects by other objects
  - keeping the positions of these objects the same
- Transpositions
  - put the objects into a different position
  - keeping the identity of these objects the same
- Both operations can be represented by permutations
  - with paying careful attention to the semantics of each permutation **and its inverse**

Is this a permutation?

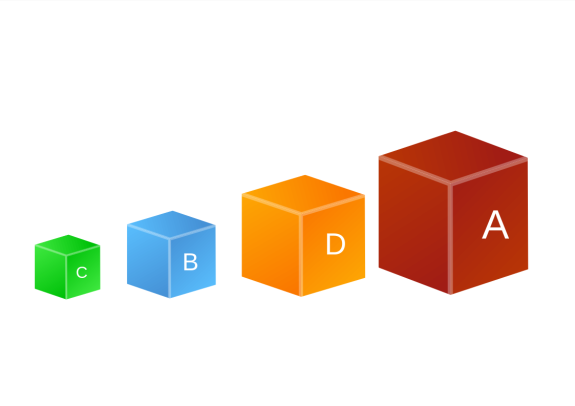


And now?



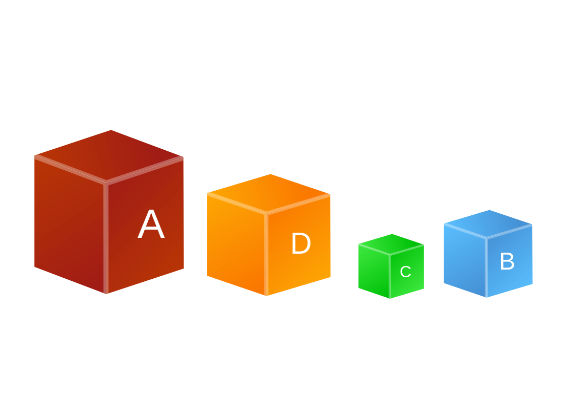
Do we order by identity, alphabetically? A<B<C<D?

Or maybe like this?



Do we order by size? tiny<small<big<huge (C<B<D<A)?

And more options...

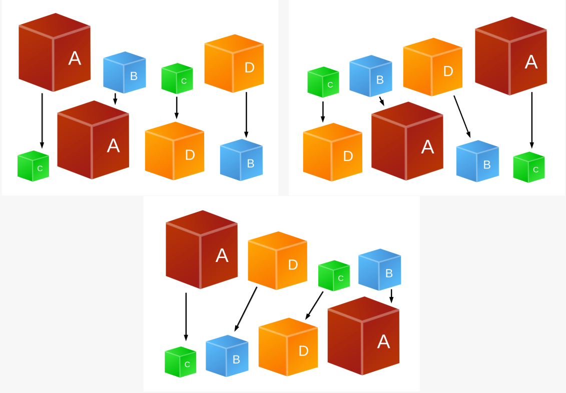


Ordered by color: red<orange<green<blue (A<D<C<B)

Case studies...

- In all of the following examples we have that
  - The position of A in the top row is the same as the position of C in the bottom row
  - The position of B in the top row is the same as the position of A in the bottom row
  - The position of C in the top row is the same as the position of D in the bottom row
  - The position of D in the top row is the same as the position of B in the bottom row
- So in some sense we always have $A \to C \to D \to B \to A$
- With the standard notation $\begin{pmatrix} A & B & C & D \\ C & A & D & B \end{pmatrix}$

Keeping position, but changing identity (1)(2)(3)



$$1 \quad 2 \quad 3 \quad 4 \qquad \begin{pmatrix} A & B & C & D \\ C & A & D & B \end{pmatrix}$$

$$\boxed{\begin{pmatrix} A & B & C & D & & & \\ C & A & D & BC & B & D & A \\ D & A & B & CA & D & C & B \\ C & B & D & A & & & \end{pmatrix}} \qquad \boxed{1 \quad 2 \quad 3 \quad 44 \quad 2 \quad 1 \quad 31 \quad 4 \quad 3 \quad 2}$$

Case studies for positions...

- We still have that
  - The position of A in the top row is the same as the position of C in the bottom row
  - The position of B in the top row is the same as the position of A in the bottom row
  - The position of C in the top row is the same as the position of D in the bottom row
  - The position of D in the top row is the same as the position of B in the bottom row
- What changes is that we look at the effect on the positions
- Also notice the direction of the arrows from bottom to top
  - which as we will see is more natural

Legacy and modern notation for positions

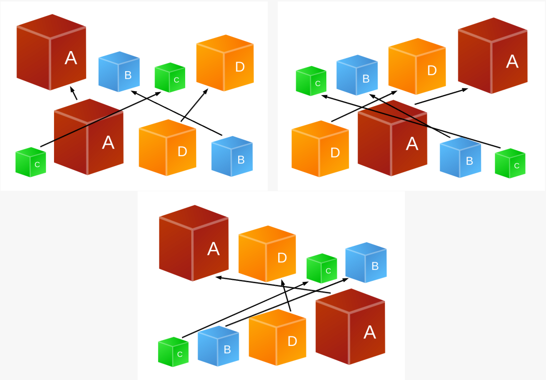
- It would be consistent to also use modern notation for positions and their permutations, like this

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 3 & 2 & 0 \end{pmatrix}$$

- But in order not to create more confusion at this stage, I will use the more standard notation like

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

Keeping identity, but changing position (1)(2)(3)



$$A \quad B \quad C \quad D \qquad \begin{pmatrix} 1 & 2 & 3 & 4 & & & \\ 3 & 1 & 4 & 21 & 2 & 3 & 4 \\ 3 & 4 & 2 & 11 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 & & & \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 2 & 4 & 1 & 3 & & & \\ 1 & 2 & 3 & 42 & 3 & 4 & 1 \\ 4 & 2 & 1 & 34 & 2 & 1 & 3 \\ 1 & 4 & 3 & 2 & & & \end{pmatrix}} \qquad \boxed{C \quad A \quad D \quad BD \quad A \quad B \quad CC \quad B \quad D \quad A}$$

Commuting diagram (before: f; after: g)

$$\begin{tikzcd}
P \arrow[r, "f"] & O \arrow[d, "\sigma"] \\
P \arrow[u, "\tau"] \arrow[r, "g"'] & O
\end{tikzcd}$$

- P is the set of positions
- O is the set of objects (or identities)
- σ is a substitution (of identity) permutation
- τ is a transposition (change of position) permutation

Permutation directions

- Consider the transformation from top (f) to bottom (g)
- $g = \sigma \circ f \circ \tau$
- $g^{-1} = \tau^{-1} \circ f^{-1} \circ \sigma^{-1}$
- Notice the direction of the permutations and their inverses
  - The substitution permutation from top (f) to bottom (g)
  - The transposition permutation from bottom (g) to top (f)
- When reversing top and bottom things turn around
  - $f = \sigma^{-1} \circ g \circ \tau^{-1}$
  - $f^{-1} = \tau \circ g^{-1} \circ \sigma$

Texts as strings or sequences of characters

$$\begin{tikzcd}
P \arrow[r, "\tau"] \arrow[rrr, bend right, "g"'] & P \arrow[r, "f"] & O \arrow[r, "\sigma"] & O
\end{tikzcd}$$

- P is n, where $n = \{0, \ldots, n-1\}$
- O is {A, B, C, ..., X, Y, Z}, our alphabet
- f is the plaintext[1] before encryption
  - or the ciphertext before decryption
- $g = \sigma \circ f \circ \tau$ is the ciphertext after encryption
  - or the plaintext after decryption

[1] Note that in this case f doesn't need to be injective nor surjective

Relation between transposition and substitution

- Even though transposition and substitution seem unrelated they are kind of dual to each other
- Both are permutations (of position, respectively identity)
- Transposition contributes to Shannon's "diffusion"
- Substitution contributes to Shannon's "confusion"
- Taken together we might[2] call them
  - **su(b)positions**
  - **trans(s)titutions**

[2] This is in no way standard or accepted terminology

Relaxing bijectivity of substitutions and transpositions

$$\begin{tikzcd} P' \arrow[r, bend right, "\tau"'] & P \arrow[l, bend right, "\tau'"'] \arrow[r, "f"] & O \arrow[r, bend right, "\sigma"'] & O' \arrow[l, bend right, "\sigma'"'] \end{tikzcd}$$

- In the case that f is the given plaintext
  - σ only needs to be injective, but not surjective
    - $id_O = \sigma' \circ \sigma$
    - O' can be “bigger” than O
  - τ only needs to be surjective, but not injective
    - $id_P = \tau \circ \tau'$
    - P' can be “bigger” than P

Expansion and compression

- Suppose that P' is indeed “bigger” than P
- We might use the following terminology if $id_P = \tau \circ \tau'$
  - τ is used for **expansion**
  - τ' is a corresponding **compression**
    - Mathematically speaking τ' is a **section** of τ

Transposition that doesn’t hide much

Internet meme (Cambridge research ???)

Aoccdrnig to a rscheearch at Cmabrigde Uinervtisy, it deosn’t mttaer in waht oredr the ltteers in a wrod are, the olny iprmoatnt tihng is taht the frist and lsat ltteer be at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit porbelm. Tihs is bcuseae the huamn mnid deos not raed ervey lteter by istlef, but the wrod as a wlohe. According to a researcher at Cambridge University, it doesn’t matter in what order the letters in a word are, the only important thing is that the first and last letter be at the right place. The rest can be a total mess and you can still read it without problem. This is because the human mind does not read every letter by itself, but the word as a whole.

(“ (sic)” deleted; “e” changed into “a”; comma inserted)

Source: `https://www.mrc-cbu.cam.ac.uk/people/matt.davis/cmabridge/`

Generating transpositions (1): Skytale

CC BY-SA 3.0, `https://commons.wikimedia.org/w/index.php?curid=1698345`

The Skytale (Scytale)

- Already used by the Spartan general Lysander
- Narrow parchment wound around a piece of wood of a given diameter
- Each side of the communication channel needs an identical piece of wood
- The length of wood is not important, as long as the message “fits”
- Encryption corresponds to a **simple columnar transposition**[3]
  - Decryption corresponds to a “row transposition”

[3] Some call this an example of a route transposition

Simple columnar (route) transposition

$$\begin{pmatrix} 0 & 1 & \cdots & c-1 \\ c & c+1 & \cdots & 2c-1 \\ \multicolumn{4}{c}{\dotfill} \\ (r-1)c & (r-1)c+1 & \cdots & rc-1 \end{pmatrix}$$

- r corresponds to the circumference (measured in letters) of the wooden stick
- The total length of the message is rc
- The plaintext is written in row by row
- The ciphertext is read out column by column

Formulas relating a text string to the rectangular block

- From rectangle to string row by row
$$(i, j) \mapsto \; ic + j$$
- From string to rectangle row by row
$$i \mapsto \; ([i/c], i \pmod c)$$
- From rectangle to string column by column
$$(i, j) \mapsto \; jr + i$$
- From string to rectangle column by column
$$i \mapsto \; (i \pmod r), [i/r])$$

Generating transpositions (2): Railfence

- A railfence cipher has as key information only the depth of the fence
  - and maybe a starting direction (down or up)
- Items are written in a zigzag pattern going down and up (or up and down) the fence
- It can be describe as an inefficient row transposition with holes (grille)
- Also, by zigzagging left and right (or right and left),
  it can be described as a columnar transposition with holes

Generating transpositions (3): Routes

There are many more ways to traverse a rectangle
or for that matter any other geometric shape

Exercise

Match Colonel Parker Hitt's methods from Figure 3.2 in Holden's book
with the route ciphers from Figure 2.2 in Hans van der Meer's syllabus

Permutation(s of positions) ciphers

- Divide the plaintext into blocks of n letters and apply
  (the same) position permutation to each block separately
- The last block is padded with "nulls"
  - The permutation maps a ciphertext position to a plaintext position

$$\begin{array}{cl} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} & \begin{array}{l} \text{cipher position} \\ \text{plain position} \end{array} \end{array}$$

  - This permutation is the same as given by the following mapping

$$\begin{array}{cl} \begin{pmatrix} 4 & 1 & 3 & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix} & \begin{array}{l} \text{cipher position} \\ \text{plain position} \end{array} \end{array} \qquad \begin{array}{cl} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} & \begin{array}{l} \text{plain position} \\ \text{cipher position} \end{array} \end{array}$$

  - **Warning: confusion! A permutation or its inverse?**
    In The Mathematics of Secrets this is also denoted by the string "4132"
- A string can be remembered by using a keyword, like "TALE"
  for the sequence 4132, using the alphabetic order

Permutation cipher compositions (products)

- Suppose the block sizes are p and q
- Then the block size of the product is given by
  - $\mathrm{lcm}(p, q)$, the least common multiple of p and q
- Hence in the case of $p = q$ you get nothing new

Keyed columnar transpositions

- These are compositions (products) of permutations and routes
  - First the permutation and then the "row-in-column-out" route
  - What happens if you do it the other way around?
- They offer a better diffusion of the plaintext throughout the whole ciphertext
- There is still no confusion though
- Shannon argued that both confusion and diffusion are important
  - We will see later how modern block ciphers achieve both

A historic transposition

Battle of Fredericksburg

Washington, D.C., November 25, 1862
To general Burnside, Falmouth, Virginia

Can Inn Ale me withe 2 oar our Ann pas Ann me flesh ends N. V. Corn Inn out with U cud Inn heaven day nest We roe Moore Tom darkey hat Greek Why Hawk of Abbott Inn B chewed I if.

From whom?

Transpositions

Adapted from slides by Hans van der Meer

Civil War

Message of president Lincoln to general major Burnside, dated Washington, November 25, 1862

BURNSIDE, Falmouth, Virginia
Can Inn Ale me withe 2 oar our Ann pas Ann me flesh ends N. V. Corn Inn out with U cud Inn heaven day nest We roe Moore Tom darkey hat Greek Why Hawk of Abbott Inn B chewed I if.

BURNSIDE, Falmouth, Virginia
If I should be in a boat off Aquia Creek at dark tomorrow, Wednesday evening, could you, without inconvenience, meet me and pass an hour or two with me? A. Lincoln

Lincoln – June 1, 1863

PLAINTEXT For Colonel Ludlow. Richardson and Brown, correspondents of the Tribune, captured at Vicksburg, are detained at Richmond. Please ascertain why they are detained and get them off if you can. The President U.S.

CRYPTOGRAM guard adam them they at wayland brown for kissing venus correspondents at neptune are off nelly turning up can get why detained tribune and times richardson the are ascertain and you fills belly this if detained please odor of ludlow commissioner

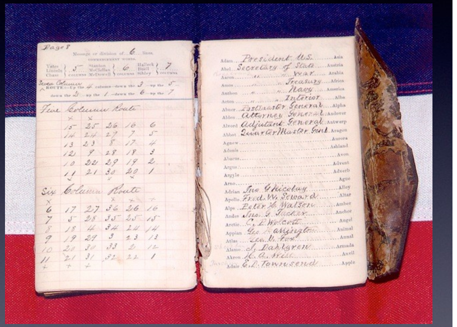
Union codebook

indicator GUARD
5x7 route tramp

null without meaning

filler on empty place

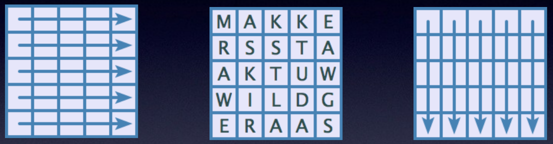
codewords
adam = President US
nelly = 4:30 pm
neptune = Richmond
odor = Vicksburg
wayland = captured
venus = colonel



Indicator GUARD

→	↓	STOP	↓	←
kissing	↓	commissioner	↓	times
for	venus	Ludlow	Richardson	and
Brown	correspondents	of	the	Tribune
wayland	at	odor	are	detained
at	neptune	please	ascertain	why
they	are	detained	and	get
them	off	if	you	can
adam	nelly	this	fills	up
↑	turning	↑	belly	↑
↑	↓	↑	←	↑
↑ START ↑	→	→	→	↑

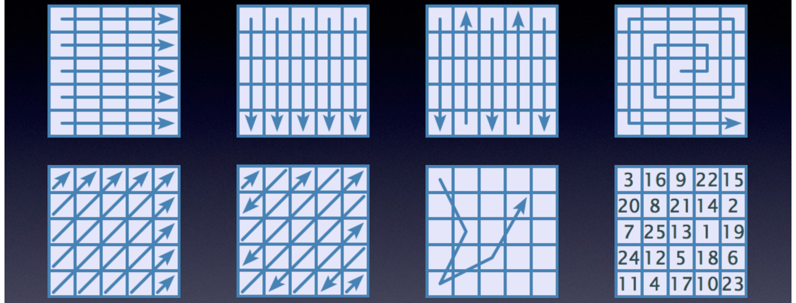
Route transposition



M	A	K	K	E
R	S	S	T	A
A	K	T	U	W
W	I	L	D	G
E	R	A	A	S

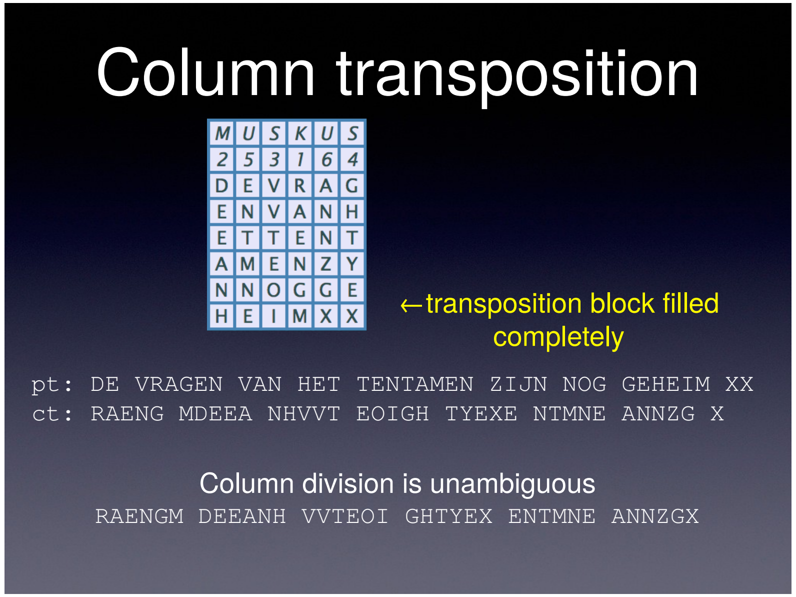
```
plaintext: MAKKERS STAAKT UW WILD GERAAS
cryptogram: MRAWE ASKIR KSTLA KTUDA EAWGS
```

Route transposition



3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

- exotic routes are knight tour, magic square
- different routes for write-in and read-out
- spiral routes show readable fragments



Column transposition

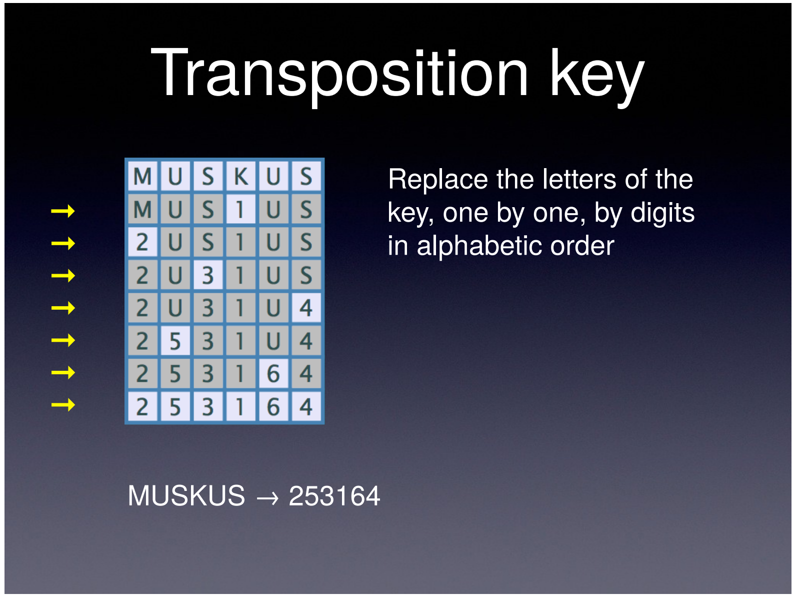
M	U	S	K	U	S
2	5	3	1	6	4
D	E	V	R	A	G
E	N	V	A	N	H
E	T	T	E	N	T
A	M	E	N	Z	Y
N	N	O	G	G	E
H	E	I	M	X	X

←transposition block filled completely

```
pt: DE VRAGEN VAN HET TENTAMEN ZIJN NOG GEHEIM XX
ct: RAENG MDEEA NHVVT EOIGH TYEXE NTMNE ANNZG X
```

Column division is unambiguous

```
RAENGM DEEANH VVTEOI GHTYEX ENTMNE ANNZGX
```

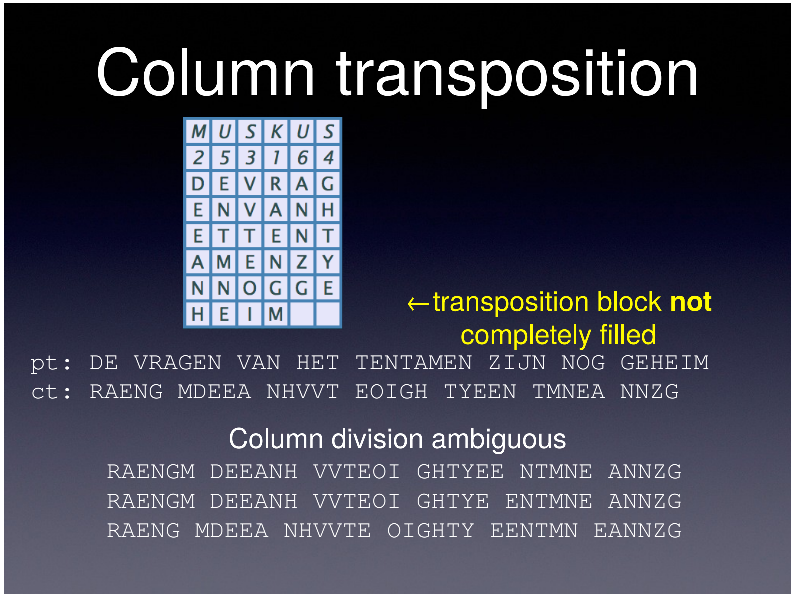


Transposition key

	M	U	S	K	U	S
→	M	U	S	1	U	S
→	2	U	S	1	U	S
→	2	U	3	1	U	S
→	2	U	3	1	U	4
→	2	5	3	1	U	4
→	2	5	3	1	6	4
→	2	5	3	1	6	4

Replace the letters of the key, one by one, by digits in alphabetic order

MUSKUS → 253164



Column transposition

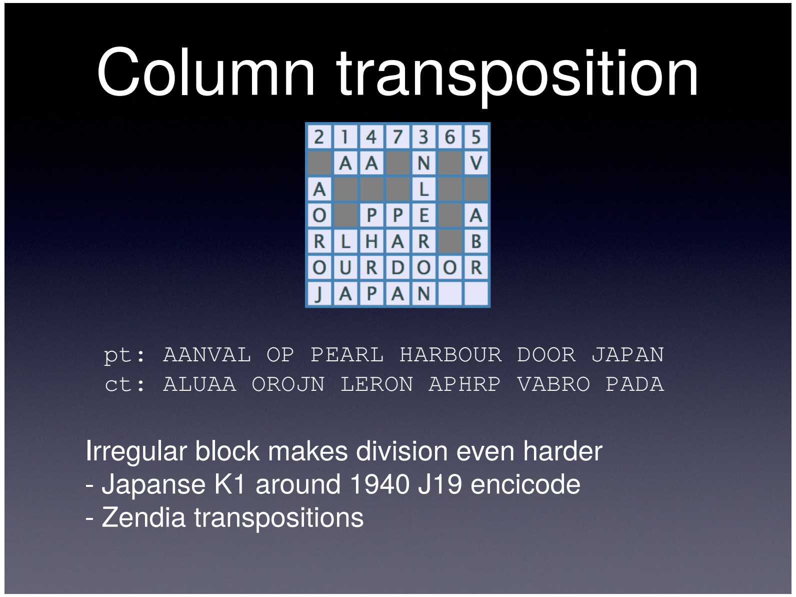
M	U	S	K	U	S
2	5	3	1	6	4
D	E	V	R	A	G
E	N	V	A	N	H
E	T	T	E	N	T
A	M	E	N	Z	Y
N	N	O	G	G	E
H	E	I	M		

←transposition block **not** completely filled

```
pt: DE VRAGEN VAN HET TENTAMEN ZIJN NOG GEHEIM
ct: RAENG MDEEA NHVVT EOIGH TYEEN TMNEA NNZG
```

Column division ambiguous

```
RAENGM DEEANH VVTEOI GHTYEE NTMNE ANNZG
RAENGM DEEANH VVTEOI GHTYE ENTMNE ANNZG
RAENG MDEEA NHVVTE OIGHTY EENTMN EANNZG
```



Column transposition

2	1	4	7	3	6	5
▨	A	A	▨	N	▨	V
A	▨	▨	▨	L	▨	▨
O	▨	P	P	E	▨	A
R	L	H	A	R	▨	B
O	U	R	D	O	O	R
J	A	P	A	N		

```
pt: AANVAL OP PEARL HARBOUR DOOR JAPAN
ct: ALUAA OROJN LERON APHRP VABRO PADA
```

Irregular block makes division even harder
- Japanse K1 around 1940 J19 encicode
- Zendia transpositions

Dubbele transpositie

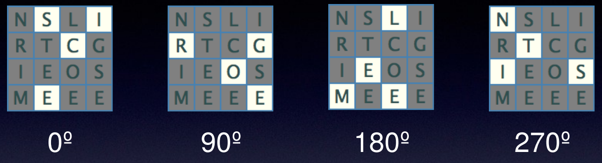
H	A	R	I	N	G	T	O	N
3	1	8	4	5	2	9	7	6
D	E	N	B	R	I	E	L	V
O	O	R	D	E	G	E	U	Z
E	N	G	E	V	A	L	L	E
N	A	L	V	A	N	A	A	R
M	A	D	R	I	D			

L	E	E	S	B	R	I	L
5	2	3	8	1	7	4	6
E	O	N	A	A	I	G	A
N	D	D	O	E	N	M	B
D	E	V	R	R	E	V	A
I	V	Z	E	R	L	U	L
A	N	R	G	L	D	E	E
L	A						

```
pt: DEN BRIEL VOOR DE GEUZEN GEVALLEN ALVA NAAR MADRID
ct: AERRL ODEVN ANDVZ RGMVU EENDI ALABA LEINE LDAOR EG
```

US Army Double Transposition

Turning grille



N	S	L	I
R	T	C	G
I	E	O	S
M	E	E	E

```
pt: SIC ERGO ELEMENTIS
ct: NSLIR TCGIE OSMEE E
```

Also called a “Fleissner grille”
Oldest: Stadtholder Willem IV in 1745
Latest: German army in 1917